How should basketball players adjust the optimal shooting angle and initial speed based according to their height and shooting distance?

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Supervision

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Introduction

Basketball, like any sport, is built on a foundation of mathematics and physics. It is probably one of the sports in which the parabolic motion is most noticeable and is more frequent during the phases of a game. The player must try to throw a ball into a basket that is placed at a much higher level than his or her own. When a basketball player throws the ball towards the basket, it traces a parabolic path. Elevating the highest point of the trajectory significantly above the basket (positioned at 3.05 meters) is crucial for improving the chances of scoring. This requires the accurate shooting angle and shooting velocity. Numerous players utilize this concept, and through effective training, they succeed in accurately shooting the basket, resulting in a higher scoring rate.

This research paper investigates the intricate interplay between the shooting height, shooting distance and shooting angles, initial velocities to project basketball into the hoop. Gaining shots is the aim of basketball game, and estimating the optimal combination of angle and velocity can significantly impact a player's performance and improve the probability of winning a competition. Utilizing physical analysis and statistical methods, this study aims to shed light on the dynamics of basketball shooting.

Huston's study [1] underscores the significant advantages of the layup shot over the straight up shot for near-the-basket shooting positions, employing a focused kinematics analysis. Another innovative paper [2] utilized three-dimensional video techniques (50 Hz) to capture basketball jump shots from short, middle, and long distances.Their findings indicate an increase in ball release speed with shooting distance, and they observe no discernible trend between shooting distance and release angle. The paper "Analysing an Optimal Angle in Basketball Free Throw" [3] aimed to reveal the relationship between the player's height with the initial velocity, optimal throwing angle and time taken for the ball to reach the hoop. The study reveals an inverse relationship between the player's height and both initial velocity and optimal throwing angle.

Methods

We will employ a projectile motion model to calculate the optimal shooting angle and initial velocity required for accurate hoop shots. It's important to note that in this analysis, we will not account for any frictional force, air resistance force, or the spinning of the basketball.

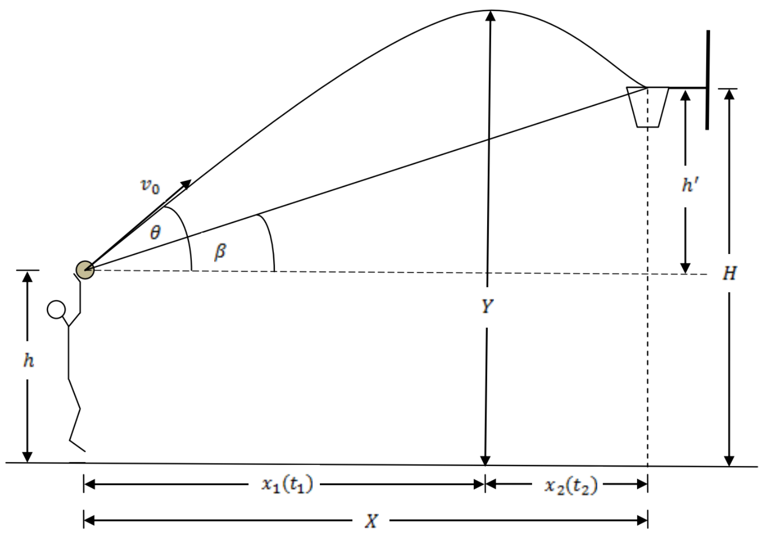


figure The Projectile Motion of Basketball Shooting

The trajectory of the ball can be characterized by the set of equations below:
Horizontal motion:

$$v_X(t) = v_0 \cos(\theta) \quad (1)$$

Horizontal Displacement:

$$X = v_0 \cos(\theta)(t_1 + t_2) \quad (2)$$

$$t = t_1 + t_2 \quad (3)$$

Vertical motion:

$$v_y(t) = v_0 \sin(\theta) - gt \quad (4)$$

where g = 9.8 m/s
At the highest point of the ball:

$$v_0 \sin(\theta) = gt_1 \quad (5)$$

The displacement in the vertical motion (h') can be expressed as:

$$h' = v_0 \sin(\theta)t - \frac{1}{2}gt^2 = (gt_1)t - \frac{1}{2}gt^2 = \frac{1}{2}g(t_1^2 - t_2^2) \quad (6)$$

where

$$h' = \frac{gt}{2} \cdot (2t_1 - t) \quad (7)$$

$$t = \frac{X}{v_0 cos(\theta)} \quad (8)$$

which is obtained in the horizontal motion, then, we plug t into the equation of h':

$$h' = \frac{g}{2} \cdot (\frac{X}{v_0 cos(\theta)})[2t_1 - \frac{X}{v_0 cos(\theta)}] = X tan(\theta) - \frac{gX^2}{2v_0^2 cos^2(\theta)} \quad (9)$$

Putting v_0 at the left side of the equation

$$v_0 = \sqrt{\frac{gX^2(\tan^2(\theta) + 1)}{2(X\tan(\theta) - h')}} \quad (10)$$

Differentiating v_0 with respect to *theta* and set the derivative equal to 0

$$\frac{dv_0}{d\theta} = X\tan(\theta) - 2h'\tan(\theta) - X = 0 \quad (11)$$

Then, we can solve for θ and v_0

$$\theta = \arctan\left(\frac{h' + \sqrt{h' + X^2}}{X}\right) \quad (12)$$

$$v_0 = g(h' + \sqrt{h'^2 + X^2})^{\frac{1}{2}} \quad (13)$$

Results and Analysis

The results of our study revealed the interplay between shooting distances, the shooting height (here we include the arm length). As the distance of shooting varies, the optimal angel and initial velocity changes. Therefore, we firstly fix the shooting distance separately at X = 1m, X = 3m, X = 6.75 m, and plot the figure of the relationship between the optimal angle and velocity as the gap between player's height and hoop changes. We set the range of the shooting height from 1.95 - 2.55 after including the arm length and the hoop length from the ground is set ay 3.05m according to the international standard.

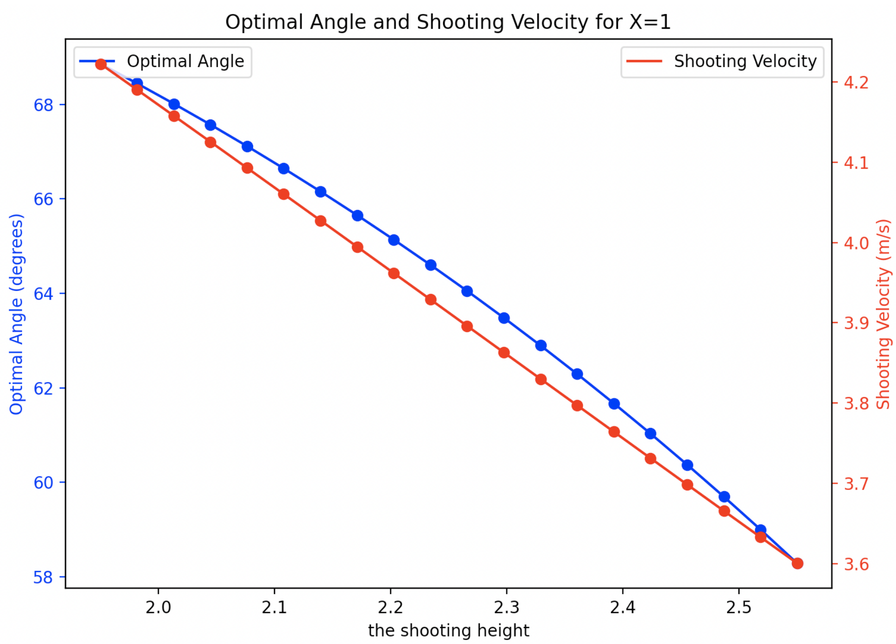


Figure 1: the short shooting distance

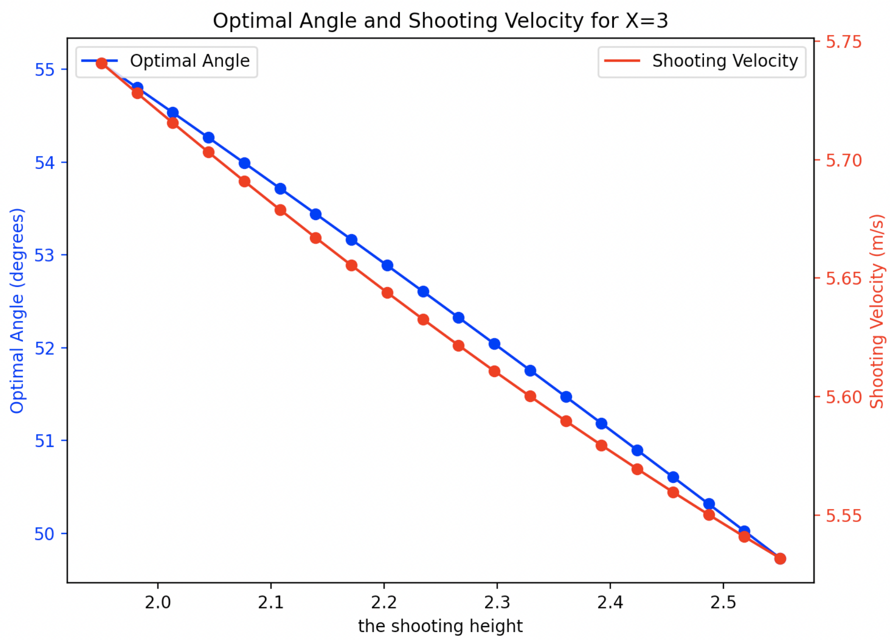


Figure 2: the middle shooting distance

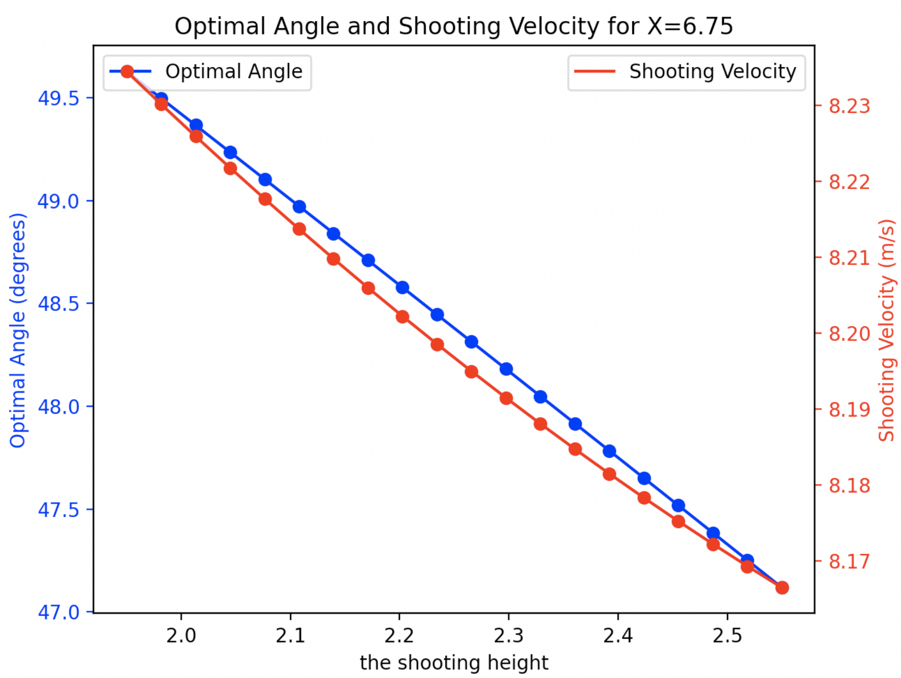
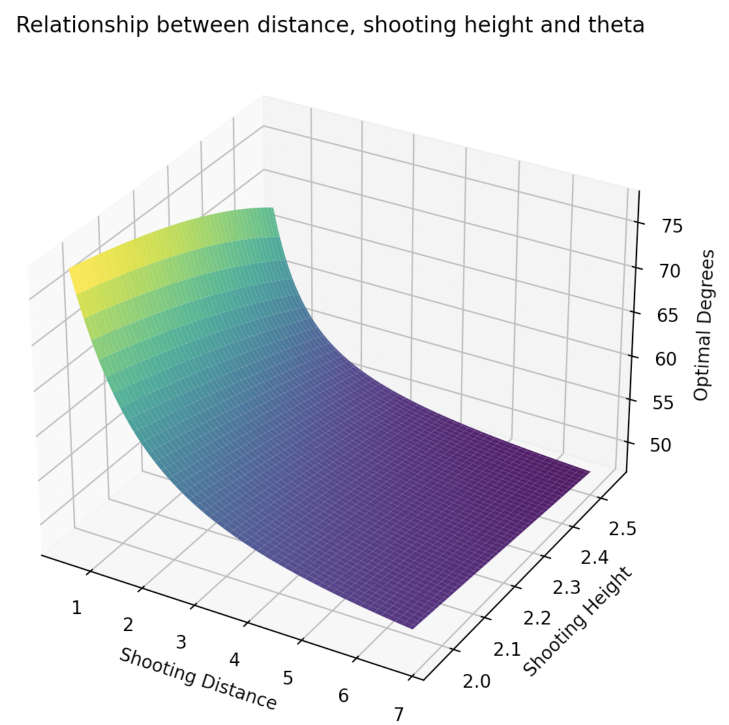


Figure 3: the far shooting distance

When the shooting position is relatively close to the hoop, which equals to 1 m, the optimal angel of shooting varies ranges from 58 degrees to 67 degrees for an adult player, and the initial shooting velocity changes from 3.6 m/s to 4.25 m/s. When the shooting position is at a middle distance to the hoop, which equals to 3 m, the optimal angel of shooting varies from 49.5 degrees to 55 degrees for an adult player, and the initial shooting velocity changes from 5.5 m/s to 5.75 m/s . When the shooting position is at the three point line, which equals to 6.75 m, the optimal angel of shooting ranges from 47 degrees to 49.5 degrees for an adult player, and the initial shooting velocity changes from 8.16 m/s to 8.24 m/s. It is noteworthy that as the shooting distance increases, the variation in shooting velocity and angle becomes smaller. The shooting velocity and optimal degree exhibit similar tendency, indicating a strong positive correlation between these two variables.

Since we have derived the formula for theta in last section and concluded that θ is a variable relate to both the gap between the basketball player, we then plotted a 3-D graph to visualize the relationship among height of shooting, distance and optimal shooting angle.

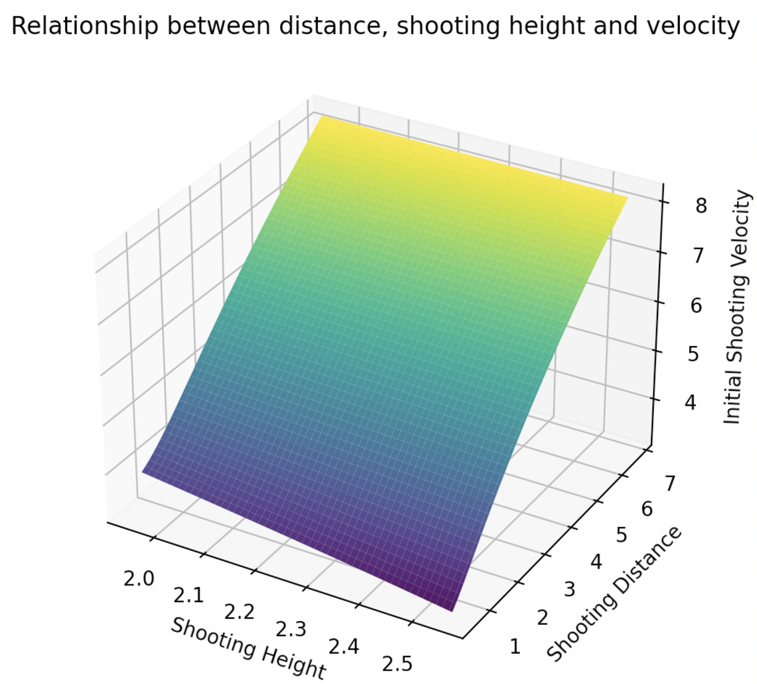


figure

3-D plot

The figure represents a slant and convex surface, leading to the visualize of the relationship among these three variables is that as shooting distance and shooting height both increases, the optimal degree for shooting decreases. Besides, the degree of inclination across the axis of shooting height of this surface is larger when the shooting distance is short, suggesting that when shooting distance is close to the hoop, varying shooting heights has a more substantial impact on the optimal shooting angle compared to the change of shooting heights with a greater shooting distance. In addition, if the shooting distance is larger than 3m, the surface tends to be flatter, which demonstrates that for the same person, his optimal shooting degree variation is larger within the 3m shooting range than that outside of 3m shooting range. Considering the end point, the optimal degree is the smallest for a distance at the three point line for the highest person.

Next, we plotted a 3-D graph to visualize the relationship among height of shooting, distance and velocity.



figure

3D plot

The figure is a slightly slanted surface, approaching nearly flat. We can draw conclusion from this figure that for the same shooting height, the far a person shoots, the larger velocity he should throw and the tangent plane indicates a roughly linear relationship between distance and initial velocity; and at a fixed shooting distance, the higher the shooting position, the less velocity is required for shooting. The impact of shooting height on initial velocity is notably smaller than that of the shooting distance on initial velocity. In addition, when a person shoots at the three point line, the height influence on the initial shooting velocity is minimal, corresponding the graph three showing that the velocity ranges from 8.16-8.24 m/s. The least required velocity for shooting is for the highest person standing at the 0.5m.

In conclusion, our visual and numerical analysis indicated that players who were able to consistently adjust their shooting angles and shooting velocities exhibited a higher overall shooting accuracy. This highlights the importance of adaptability in a player's shooting technique.

Conclusion

In conclusion, my study examined the interplay among shooting distances, shooting height (including arm length), optimal shooting angles, and initial velocities in the context of basketball shooting. The following key findings emerged from our investigation:

1. Shooting Distance Impact: A larger shooting distance requires a smaller shooting angel and larger shooting velocity. The ideal shooting angle range for distance at X = 1m, X = 3m, X = 6.75m are correspondingly 58°-68.5°, 48.5°-55.1° and 47°-49.7°. The ideal shooting velocity range for distance at X = 1m, X = 3m, X = 6.75m are correspondingly 3.6 - 4.25 m/s, 5.55-5.75 m/s and 8.16/8.24 m/s.

2. Height Impact: at the same distance, for a shorter person, the optimal degree for shooting needs to be larger and the initial velocity should be faster.

Coaches and players can leverage these findings to tailor training regimens based on individual height and shooting distances. Adjusting shooting angles and velocities according to the findings will lead to more consistent and accurate shooting performances. Besides, the traditional statement that "Basketball is just the game for high people" is not grounded according to the finding, as long as the player masters the law of projectile motion in basketball, he can improve the accuracy of shooting at the basket. The findings underscore the importance of adaptability in a player's shooting technique, emphasizing the need for players to adjust their shooting angle and velocity based on varying game scenarios. This research contributes to the ongoing exploration of optimized shooting strategies for enhanced performance in basketball.

Future research could explore additional factors, such as air forces, frictional force and direction of rotation of the basketball to create a more comprehensive model for optimizing basketball shooting. Additionally, investigating the impact of these findings in live-game situations and exploring adaptive learning algorithms for real-time adjustments could further enhance the accuracy of shooting.

Appendix

```python
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

def get_theta(X, height_p):
    #
    h_prime = H - height_p
    inner_expression = ((h_prime + np.sqrt(np.array(h_prime) ** 2 + np.array(X) ** 2)) / np.array(X))
    #print('inner', inner_expression)
    theta = np.arctan(inner_expression)
    theta_deg = np.degrees(theta)
    #print(theta)
    return theta_deg
def get_v0(X, height_p):
    g = 9.8
    h_prime = H - height_p

    v0 = np.sqrt(g * (h_prime + (h_prime ** 2 + X ** 2)) ** (1/2))
    return v0

# Press the green button in the gutter to run the script.
if __name__ == '__main__':
    start_value = 1.95
    end_value = 2.55
    step_size = 0.05
    height_p = np.linspace(start_value, end_value, 100)
    H = 3.05
    h_prime = H - height_p
    X = np.linspace(0.5, 6.75, 100)
    # Create 2D arrays from X, h_prime, and Z using meshgrid
    X, height_p= np.meshgrid(X, height_p)
    Z = get_v0(X, height_p)
```

```python
    # 三维图形
    fig = plt.figure()
    ax = fig.add_subplot(111, projection='3d')
    ax.plot_surface(height_p,X, Z, cmap='viridis')
    ax.set_xlabel('Shooting Height')
    ax.set_ylabel('Shooting Distance')
    ax.set_zlabel('Initial Shooting Velocity')  # Update z-label to indicate degrees
    ax.set_title('Relationship between distance, shooting height and velocity')
    plt.show()
```

figure code

References

[1] R. L. Huston and C. A. Grau, "Basketball shooting strategies — the free throw, direct shot and layup," *Sports Engineering*, vol. 6, no. 1, pp. 49–64, March 2003. [Online]. Available: https://doi.org/10.1007/BF02844160

[2] S. Miller and R. M. Bartlett, "The effects of increased shooting distance in the basketball jump shot," *Journal of Sports Sciences*, vol. 11, no. 4, pp. 285–293, 1993, pMID: 8230388. [Online]. Available: https://doi.org/10.1080/02640419308729998

[3] S. M. Nor-Al-Din, N. N. S. Shamsuddin, R. N. Khairiah, and N. M. Sukri, "Analysing an optimal angle in basketball free throw," *Journal of Physics: Conference Series*, vol. 2084, no. 1, p. 012017, nov 2021. [Online]. Available: https://dx.doi.org/10.1088/1742-6596/2084/1/012017